

# Channel Capacity for Flat Fading MIMO System with Different Distributions: Analysis and Simulation

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## Abstract

Multi-input multi-output (MIMO) antenna system becomes one of the hottest research topics in the communication field during the past few years due to its obvious edge over simple single antenna systems. This paper presents analyses and simulations for the effects of the channel distribution on the overall channel capacity for a MIMO system under flat fading. Several types of distributions (Gaussian, Poisson, and Rayleigh) are considered to generate the channel matrix and determine the capacity for several cases of transmits and receiver antennas.

## Keywords

MIMO; Channel Matrix; Capacity; Gaussian Distribution; Poisson Distribution; Rayleigh Distribution; Fading

## Introduction

There are many factors that effect on the transmitted signal passing through communication channels that cause error in received signal comparing to that original transmitted one. These factors can be divided into deterministic in nature (such as linear and nonlinear distortion, intersymbol interference, etc.) and nondeterministic (such as addition of noise, multipath fading, ect.). The performance and capacity of communication system (usually measured by bit error rate and data rate respectively) are experience degradation due to these imperfections.

Many valuable solutions have been proposed and implemented to overcome one or more of these imperfections. One of these solutions is the use of multi-input multi-output (MIMO) system that can be used to improve system performance and capacity in fading channels. During the past few years until present, many considered research efforts are conducted to deal with the MIMO system characteristics and applications. The multiple antennas

can be used to increase data rates through multiplexing or to improve performance through diversity. MIMO system utilizes multiple antennas at the transmitter and receiver.

The use of multiple antennas at both the transmitter and the receiver can simply be seen as a tool to further improve the signal-to-noise/interference ratio and/or achieve additional diversity against fading, compared to the use of only multiple receive antennas or multiple transmit antennas. However, in the case of multiple antennas at both the transmitter and the receiver, there is also the possibility for so-called spatial multiplexing, allowing for more efficient utilization of high signal-to-noise/interference ratios and significantly higher data rates over the radio interface.

In this work, the effect of multipath flat fading on the signal and how the matrix channel model distribution effect on capacity are considered. The achieved results show that the choice of channel distribution is a critical parameter in the sense of expected capacity and can be led to better modeling for different operation scenarios. The rest of the paper is outlined as follows: section 2 presents a theoretical background for the main considered distribution in the communication channels. Section 3 illustrates the achieved simulation results, then the main achieved points from this study are given in section 4.

## Theory

Referring to a general MIMO system showing in Fig. 1 with  $N_T$  transmits antennas and  $N_R$  receiver antennas. The signal model is represented as:

$$r = Hx + n \quad (1)$$

where  $r$  is  $(N_R \times 1)$  received signal vector,  $x$  is  $(N_T \times 1)$

transmitted signal vector,  $\mathbf{n}$  is  $(N_T \times 1)$  complex additive white Gaussian noise (AWGN) vector with variance equal to  $\sigma$ , and  $\mathbf{H}$  is the  $(N_R \times N_T)$  channel matrix.

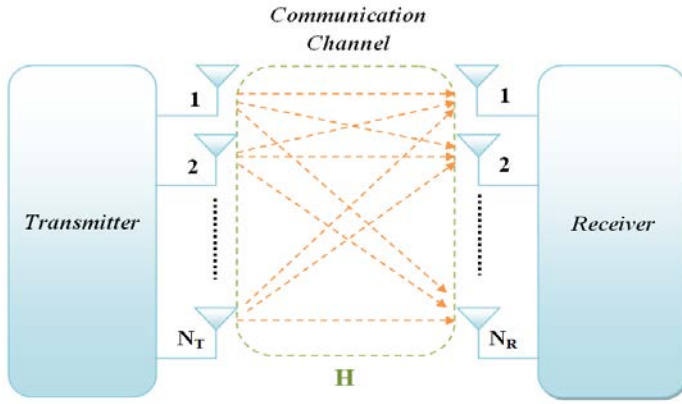


FIG. 1 GENERAL MIMO SYSTEM MODEL

The channel matrix  $\mathbf{H}$  represents the effect of the medium on the transmitter –receiver links. The channel matrix  $\mathbf{H}$  is describe as:

$$\mathbf{H} = \begin{bmatrix} h_{11} & \cdots & h_{1N_T} \\ \vdots & \ddots & \vdots \\ h_{N_R 1} & \cdots & h_{N_R N_T} \end{bmatrix} \quad (2)$$

Channel matrix may offer  $K$  equivalent parallel subchannel with different mean gains, where

$$K = \text{RANK}(\mathbf{H}\mathbf{H}^H) \leq \text{MIN}(N_T, N_R) \quad (3)$$

Singular Value Decomposition (SVD) simplification can be used to demonstrate the effect of channel matrix  $\mathbf{H}$  on the capacity. Thus, channel matrix  $\mathbf{H}$  can be expressed as:

$$\mathbf{H} = \mathbf{U}\mathbf{B}\mathbf{V}^H \quad (4)$$

With the columns of the unitary matrix  $\mathbf{U}$  ( $N_R \times N_R$ ) containing the eigenvectors of  $\mathbf{H}\mathbf{H}^H$  and the columns of the unitary matrix  $\mathbf{V}$  ( $N_T \times N_T$ ) containing the eigenvectors of  $\mathbf{H}^H\mathbf{H}$ . The diagonal matrix  $\mathbf{B}$  ( $N_R \times N_T$ ) has nonnegative, real valued elements (called singular values) equal to the square roots of the Eigen values  $\lambda$  of  $\mathbf{H}\mathbf{H}^H$ .

Assuming that the channel is known at both Tx and Rx (full or prefect channel sensing information CSI) then the maximum normalized capacity with respect to the bandwidth (in term of b/s/Hz spectral efficiency) of parallel subchannels equals:

$$C = \sum_{i=1}^K \log_2 \left( 1 + \lambda_i \frac{P_i}{\sigma_N^2} \right) \quad (5)$$

where  $P_i$  is the power allocated to each subchannel  $i$  and can be determined to maximize the capacity using

water filling theorem such that each subchannel was filled up to a common level  $D$ :

$$\frac{1}{\lambda_1} + P_1 + \cdots + \frac{1}{\lambda_k} + P_k = D \quad (6)$$

or

$$P_i = D - \frac{1}{\lambda_i} \quad (7)$$

Such that it satisfies the following condition that sums of all subchannels power equal to the total transmitted power or :

$$\sum_{i=1}^K P_i = P_{TX} \quad (8)$$

and if  $\frac{1}{\lambda_i} > D$  then  $P_i$  is set to zero.

A brief overview of the random distributions used in this work is listed as:

### Gaussian Distribution

One of the most important probability distributions is the Gaussian (or normal) distribution. The density function for this distribution is given by:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (9)$$

where  $\mu$  is the mean,  $\sigma$  is the standard deviation, and  $\sigma^2$  is the variance. This distribution is used to generate the channel matrix and determine the related capacity for the system:

$$\mathbf{H}_{\text{Gaussian}} = \begin{bmatrix} h_{11} & \cdots & h_{1N_T} \\ \vdots & \ddots & \vdots \\ h_{N_R 1} & \cdots & h_{N_R N_T} \end{bmatrix} \quad (10)$$

### Poisson Distribution

In practice Poisson random variables arise when events occur in time or space. In such a way that the average number of events occurring in a given time or space interval of fixed length is constant and events occurring in successive intervals are independent. The Poisson distribution is often seen as a good model of rare events. Poisson distribution depends on only one positive value called the parameter of the Poisson ( $\lambda$ ).

The Poisson probability function is given as:

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad (11)$$

This distribution is used to generate the channel matrix and determine the related capacity for the system:

$$\mathbf{H}_{\text{Poisson}} = \begin{bmatrix} h_{11} & \cdots & h_{1N_T} \\ \vdots & \ddots & \vdots \\ h_{N_R 1} & \cdots & h_{N_R N_T} \end{bmatrix} \quad (12)$$

### Rayleigh Distribution

The Rayleigh distribution is mainly used in communication engineering and it may be considered as a special case of the chi-squared distribution. The Rayleigh distribution is also a special case of the two-parameter Weibull distribution. The probability density function for this distribution is given by:

$$f(x) = \frac{x}{\delta^2} e^{-\frac{x^2}{2\delta^2}} \quad (13)$$

where  $\delta$  is the scale parameter of Rayleigh distribution.

This distribution is used to generate the channel matrix and determine the related capacity for the system:

$$H_{\text{Rayleigh}} = \begin{bmatrix} h_{11} & \cdots & h_{1N_T} \\ \vdots & \ddots & \vdots \\ h_{N_R 1} & \cdots & h_{N_R N_T} \end{bmatrix} \quad (14)$$

### Simulation Results

In this work, MATLAB m-file is used to model and simulate the effects of several types of distributions (Gaussian, Poisson, and Rayleigh) for a MIMO system under flat fading to generate the channel matrix.

Water filling theorem with its concept is considered to determine the power allocation for the equivalent parallel subchannel and determine the capacity for a wide range of SNR (-10 dB to 30 dB) with a resolution step of 2dB and noise equal to 0.0001. The simulation is done for several pairs of  $N_R$  and  $N_T$  as detailed in Table 1.

TABLE 1 NUMBER OF TRANSMITTER AND RECEIVER ANTENNAS

Case No.	Number of transmitter antennas ( $N_T$ )	Number of receiver antennas ( $N_R$ )
1 <sup>st</sup>	1	1
2 <sup>nd</sup>	2	2
3 <sup>rd</sup>	3	2
4 <sup>th</sup>	2	3
5 <sup>th</sup>	4	4
6 <sup>th</sup>	5	5
7 <sup>th</sup>	6	6

### Gaussian Distribution

The first considered distribution is Gaussian (normal) distribution; with zero mean ( $\mu$ ) and unity standard deviation ( $\sigma$ ). The achieved results are shown in Fig. 2.

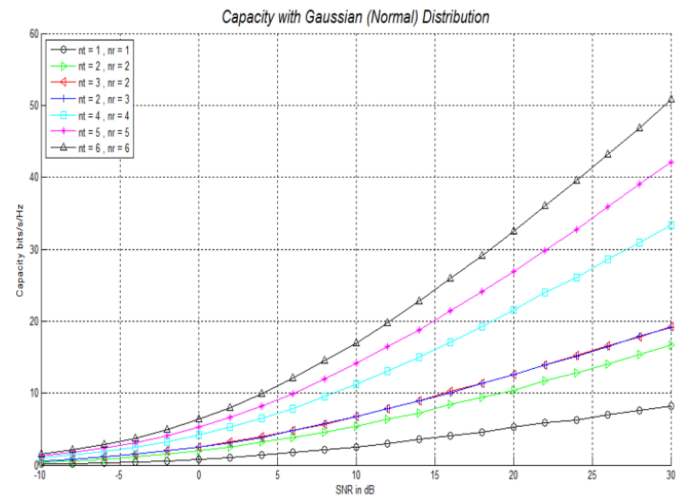


FIG. 2 CAPACITY WITH GAUSSIAN (NORMAL) DISTRIBUTION

The capacity of the system (in term of b/s/Hz) is calculated for each corresponding case illustrated in Table 1 over a wide range of SNR (-10 dB to 30 dB). Each of the seven cases is represented with capacity curves using different colors and special marker symbols as shown in the upper left corner of the Fig. 2.

From examination of the 1<sup>st</sup> curve ( $N_T = 1$ ,  $N_R = 1$ ), it's noticeable that the capacity is increased as SNR increases with respect to Eq. (5) which is coincides with the generating channel matrix  $H$  (its single element in this case) by Gaussian distribution as in Eq. (9).

From examination of the 2<sup>nd</sup> curve ( $N_T = 2$ ,  $N_R = 2$ ), it's noticeable that the capacity is increased for the same value of SNR compared to the first considered case because of the increase in number of antennas in both transmitter and receiver sides. The capacity increasing is corresponded to the  $H_{\text{Gaussian}}$  (its 2x2 matrix in this case) in approximately exponential manner.

The results listed in the 3<sup>rd</sup> curve corresponds to  $N_T = 3$  and  $N_R = 2$  shows that the capacity increases for the same value of SNR compared to the first and second cases as approximating exponential behavior.

The 4<sup>th</sup> case ( $N_T = 2$ ,  $N_R = 3$ ) is almost the same as that for the previous ones, with a slight difference, to that in the third case. This represents the reverse antenna allocation in transmitter and receiver sides comparing to the 3<sup>rd</sup> case. This case is added to explain the effect on capacity for the same number of antenna but with different antenna allocation between the transmitter and receiver.

In the 5<sup>th</sup> case ( $N_T = 4$ ,  $N_R = 4$ ), it's clear that the capacity still increased compared to the previous cases

even for small SNR and that the increase is become more and more approximating the exponential behavior. These observations are still similar to the 6<sup>th</sup> and 7<sup>th</sup> cases.

Refereeing to the cases (1-7), it's clear that the increase of capacity is related to increasing number of antennas in both transmitter and receiver sides and there amount allocation that increase the channel matrix elements represents the gain of each path. This capacity increase is also governed by the Gaussian distribution which is used to approximate the operation conditions of the system and generating H.

### Poisson Distribution

The second considered case is Poisson distribution; with unity parameter ( $\lambda$ ) which represents both the mean and the variance of the distribution. The achieved results are shown in Fig. 3.

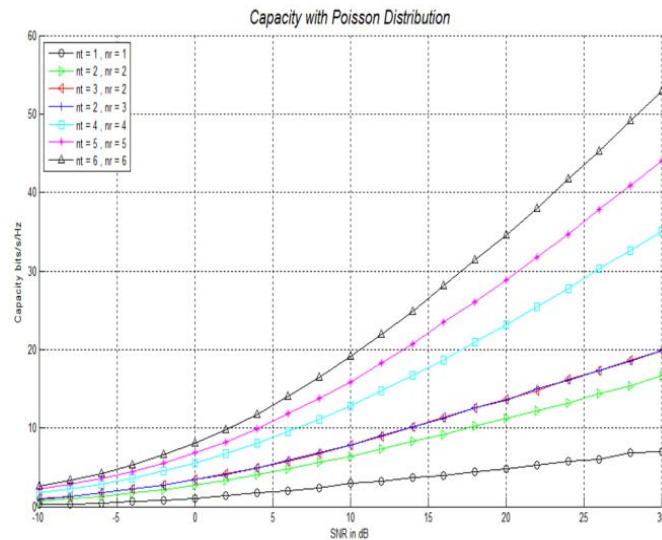


FIG. 3 CAPACITY WITH POISSON DISTRIBUTION

The capacity of the system (in term of b/s/Hz) is calculated for each case listed in Table 1 over a wide range of SNR (-10 dB to 30 dB). Each of the seven cases is represented with capacity curves using different colors and special marker symbols clarified in the upper left corner of Fig. 3. The observations of Poisson distribution shows that, in general, the capacity increases with growing number of antennas in both sides, i.e., in the transmitter and receiver sides. This is still similar to that of Gaussian (standard normal) distribution. However, it's noticeable that the capacity with Poisson distribution is lower compared to capacity with Gaussian distribution for the 1<sup>st</sup> case while it's higher in value compared to the Gaussian distribution for the other cases (2-7).

### Rayleigh Distribution

The third case considered in this work is Rayleigh distribution; with unity scale parameter  $\delta$ . The achieved results are shown in Fig. 4.

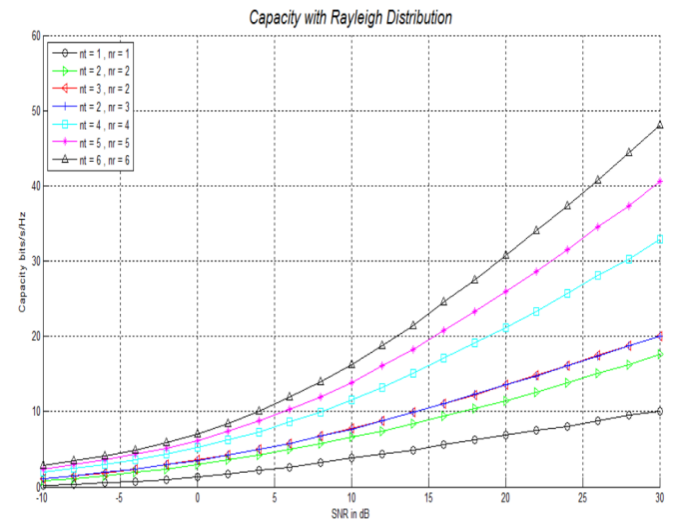


FIG. 4 CAPACITY WITH RAYLEIGH DISTRIBUTION

As for both Gaussian and Poisson distributions, the capacity of the system (in term of b/s/Hz) is calculated for each case listed in Table 1 over a wide range of SNR (-10 dB to 30 dB). Each of the seven cases is represented with capacity curves using different colors and special marker symbols as shown in Fig. 4. The results for this distribution show that the capacity is increasing with number of antennas in both transmitter and receiver sides. This also has a similar behavior to that of the two other distributions, but Fig. 4 shows that, as SNR increases the capacity with Rayleigh distribution for the cases (1,2) is higher compared to the capacity with Poisson distribution while it is lower than that for other cases (5-7) and almost equal for case (3, 4).

Also, by comparison of the achieved results shown in Fig. 4 with that in Fig. 2, the capacity with Rayleigh distribution for the cases (1-4) is higher compared to the capacity with Gaussian distribution while it is lower for the other cases (5-7).

### Conclusions

The achieved results show that the selection of channel distribution gives an impact over the expected capacity of the MIMO system. It can be seen that when channel matrix considered being Rayleigh distribution, the expected capacity is higher for small number of antenna at Tx and Rx compared to that with Poisson and Gaussian distributions. This tends to become the

lowest compared to them as the number of  $T_x$  and  $R_x$  increases for the same amount of SNR.

In the case of Poisson distribution, the capacity is lower compared to the other two distributions for small number of antennas at  $T_x$  and  $R_x$ . This becomes the highest compared to them as the number of  $T_x$  and  $R_x$  increases for the same amount of SNR.

In the case of Gaussian distribution, the capacity is higher compared to the Poisson distribution and lowers than that with Rayleigh distribution for small number of antenna at  $T_x$  and  $R_x$ . This behavior is reversed when the number of  $T_x$  and  $R_x$  increases for the same amount of SNR.

These results provide a better understanding of the effect of each distribution and how it can be used to approximate different environments. Also, the investigation of more channel distributions will lead to better modeling of channel for different operation scenarios and various environments.

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